Chapter 12 Solute and Solutions

1. Concentration units
2. Principles of solubility
3. **Colligative properties** of nonelectrolytes
4. Colligative properties of electrolytes

Modified by Dr. Cheng-Yu Lai
Solute

A solute is the dissolved substance in a solution.

- Salt in salt water
- Sugar in soda drinks
- Carbon dioxide in soda drinks

Solvent

A solvent is the dissolving medium in a solution.

- Water in salt water
- Water in soda

Solute + Solvent = Solution
Calculations of Solution Concentration: Mass Percent

Mass percent is the ratio of mass units of solute to mass units of solution, expressed as a percent

\[
\text{Mass percent} = \left( \frac{\text{mass of solute}}{\text{mass of solution}} \right) \times 100
\]
Calculations of Solution Concentration

**Mole fraction** – the ratio of moles of solute to total moles of solution

\[
\text{Mole fraction of } A = \chi_A = \frac{n_A}{n_A + n_B}
\]

A solution contains 1 mole of water and 4 mole of ethanol. The mole fraction of water and ethanol will be
(a) 0.2 water + 0.8 ethanol  (b) 0.4 water + 0.6 ethanol  (c) 0.6 water + 0.8 ethanol  
(d) 0.8 water + 0.2 ethanol

Ans: (a)
Calculations of Solution Concentration: Molarity

**Molarity** is the ratio of moles of solute to liters of solution

\[ \text{Molarity} = M = \frac{\text{moles of solute}}{\text{Liter of solution}} \]

Suppose you had 2.00 moles of solute dissolved into 1.00 L of solution. What's the molarity?

The answer is 2.00 M.

\[ \text{Molarity} = \frac{2.00 \text{ mol}}{1.00 \text{ L}} \]
**Preparation of Molar Solutions**

Problem: How many grams of sodium chloride are needed to prepare 1.50 liters of 0.500 M NaCl solution?

- **Step #1:** Ask “How Much?” (What volume to prepare?)
- **Step #2:** Ask “How Strong?” (What molarity?)
- **Step #3:** Ask “What does it weigh?” (Molar mass is?)

\[
\begin{array}{c|c|c|c}
1.500 \text{ L} & 0.500 \text{ mol} & 58.44 \text{ g} \\
1 \text{ L} & 1 \text{ mol} & 1 \text{ mol} = 43.8 \text{ g}
\end{array}
\]
Dilution

Problem: What volume of stock (11.6 M) hydrochloric acid is needed to prepare 250 mL of 3.0 M HCl solution?

\[ (11.6 \text{ M})(x \text{ Liters}) = (3.0 \text{ M})(0.250 \text{ Liters}) \]

\[ x \text{ Liters} = \frac{(3.0 \text{ M})(0.250 \text{ Liters})}{11.6 \text{ M}} \]

\[ = 0.065 \text{ L} \]
Calculations of Solution Concentration

**Molality** – moles of solute per kilogram of solvent

\[
\text{Molality } m = \frac{\text{moles solute}}{\text{ki log ram solvent}}
\]

Suppose you had 2.00 moles of solute dissolved into 1.00 L of solvent. What's the molality?

The answer is 2.00 m.

\[
\text{Molality } = \frac{2.00 \text{ mol}}{1.00 \text{ kg}}
\]
Units of Concentration

Molarity (M) = \( \frac{\text{Moles of solute}}{\text{Liters of solution}} \)

Mole fraction (\( X \)) = \( \frac{\text{Moles of component}}{\text{Total moles making up solution}} \)

Mass percent = \( \frac{\text{Mass of component}}{\text{Total mass of solution}} \times 100\% \)

Molality (\( m \)) = \( \frac{\text{Moles of solute}}{\text{Mass of solvent (kg)}} \)
Example 12.4 Calculating Concentrations

A solution is prepared by dissolving 17.2 g of ethylene glycol (C\textsubscript{2}H\textsubscript{6}O\textsubscript{2}) in 0.500 kg of water. The final volume of the solution is 515 mL. For this solution, calculate the concentration in each unit.

a. molarity
b. molality
c. percent by mass
d. mole fraction
e. mole percent

Solution

a. To calculate molarity, first find the amount of ethylene glycol in moles from the mass and molar mass. Then divide the amount in moles by the volume of the solution in liters.

\[
mol \text{ C}_2\text{H}_6\text{O}_2 = \frac{17.2 \text{ g C}_2\text{H}_6\text{O}_2}{62.07 \text{ g C}_2\text{H}_6\text{O}_2} = 0.2771 \text{ mol C}_2\text{H}_6\text{O}_2
\]

\[
\text{Molarity (M)} = \frac{\text{amount solute (in mol)}}{\text{volume solution (in L)}} = \frac{0.2771 \text{ mol C}_2\text{H}_6\text{O}_2}{0.515 \text{ L solution}} = 0.538 \text{ M}
\]
A solution is prepared by dissolving 17.2 g of ethylene glycol (C₂H₆O₂) in 0.500 kg of water. The final volume of the solution is 515 mL. For this solution, calculate the concentration in each unit.

a. molarity  

b. molality  

c. percent by mass  

d. mole fraction  

e. mole percent  

b. To calculate molality, use the amount of ethylene glycol in moles from part (a), and divide by the mass of the water in kilograms.

\[
\text{Molality (} m \text{)} = \frac{\text{amount solute (in mol)}}{\text{mass solvent (in kg)}}
\]

\[
= \frac{0.2771 \text{ mol } C_2H_6O}{0.500 \text{ kg } H_2O}
\]

\[
= 0.554 \text{ m}
\]

c. To calculate percent by mass, divide the mass of the solute by the sum of the masses of the solute and solvent and multiply the ratio by 100%.

\[
\text{Percent by mass} = \frac{\text{mass solute}}{\text{mass solution}} \times 100\%
\]

\[
= \frac{17.2 \text{ g}}{17.2 \text{ g} + 5.00 \times 10^2 \text{ g}} \times 100\%
\]

\[
= 3.33\%
\]
Example 12.4  Calculating Concentrations

Continued

d. To calculate mole fraction, first determine the amount of water in moles from the mass of water and its molar mass. Then divide the amount of ethylene glycol in moles (from part (a)) by the total number of moles.

\[
mol\ H_2O = 5.00 \times 10^2\ g\ H_2O \times \frac{1\ mol\ H_2O}{18.02\ g\ H_2O} = 27.75\ mol\ H_2O
\]

\[
\chi_{solute} = \frac{n_{solute}}{n_{solute} + n_{solvent}} = \frac{0.2771\ mol}{0.2771\ mol + 27.75\ mol} = 9.89 \times 10^{-3}
\]

e. To calculate mole percent, multiply the mole fraction by 100%.

\[
\text{mol}\% = \chi_{solute} \times 100\% = 0.989\%
\]

For Practice 12.4

A solution is prepared by dissolving 50.4 g sucrose \((C_{12}H_{22}O_{11})\) in 0.332 kg of water. The final volume of the solution is 355 mL. Calculate the concentration of the solution in each unit.

a. molarity  
b. molality  
c. percent by mass  
d. mole fraction  
e. mole percent
Solubility Factors

The extent to which a solute dissolves in a solvent depends on several factors:

1. The nature of **solvent and solute particles and the interaction between them**
2. The *temperature* at which the solution forms
3. The *pressure*, in cases of gaseous solutes
1. Interaction: For Polar/Non-Polar Molecules Like Dissolves in like

- **Like dissolves like**
  - Polar solutes dissolve in polar solvents
  - Nonpolar solutes dissolve in nonpolar solvents

- **Nonpolar substances have poor affinity for water**
  - Petroleum
  - Hydrocarbons (pentane, \( \text{C}_5\text{H}_{12} \))

- **Polar substances dissolve easily in water**
  - Alcohols, \( \text{CH}_3\text{OH} \)
  - Solubility of alcohols decreases as the molar mass of the alcohol increases
Applications of “Like Dissolves Like” Concept

Nonpolar solutes dissolve best in nonpolar solvents

- Fats
- Steroids
- Waxes

Benzene
Hexane
Toluene

Polar and ionic solutes dissolve best in polar solvents

- alcohols
- Water
Solubility Trends

- The solubility of MOST solids increases with temperature.
- The rate at which solids dissolve increases with increasing surface area of the solid.
- The solubility of gases decreases with increases in temperature.
- The solubility of gases increases with the pressure above the solution.
Therefore...

**Solids** tend to dissolve best when:
- Heated
- Stirred
- Ground into small particles

**Gases** tend to dissolve best when:
- The solution is cold
- Pressure is high

Gas Evolving @ RT more than ice
2. Temperature and Solubility

• When a solute dissolves, equilibrium is established
  – \( \text{NaNO}_3 (s) \rightleftharpoons \text{Na}^+ (aq) + \text{NO}_3^- (aq) \)
  – \( \text{O}_2 (g) \rightleftharpoons \text{O}_2 (aq) \)
• An *increase in temperature* always shifts the equilibrium to *favor an endothermic process* (Ch12.)
  – Dissolving a *solid in a liquid* is usually an *endothermic process*
  – Solubility tends to *increase with temperature* for most solids

Temp ↑ Solid Solubility ↑
Example: Temperature and the Solubility of a Gas

• For a gas, the dissolution process is **exothermic**, so the reverse process (**gas evolving from solution**) is **endothermic**

• Therefore, for gases, **solubility decreases with increasing temperature** – Ch12 Le Charliter’s Principle

![Image of gas evolving at room temperature compared to ice](image)
3. Pressure and the Solubility of a Gas

• Pressure has a major effect on the solubility of a gas in a liquid, but little effect on other systems.

• At low to moderate pressure, the concentration of a gas increases with the pressure (Henry’s law):
  
  \[ C_g = kP_g, \]

  where

  • \( P_g \) is the partial pressure of the gas over the solution
  • \( C_g \) is the concentration of the gas
  • \( k \) is a constant (the Henry’s Law constant)

\[ C = kP \]
Figure 10.6 – Solubility Effects: Pressure and Temperature

Graph (a): Solubility of [O₂] as a function of Temperature (°C).

Graph (b): Solubility of [O₂] as a function of Pressure (atm).
What pressure of CO\(_2\) is required to keep the [CO\(_2\)] = 0.12 M at 25°C?

<table>
<thead>
<tr>
<th>Given:</th>
<th>(S = [\text{CO}_2] = 0.12 \text{ M},)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find:</td>
<td>(P ) of CO(_2), atm</td>
</tr>
</tbody>
</table>

Concept Plan:  
\[
P = \frac{S}{k_H}
\]

Relationships:  
\(S = k_H P, \ k_H = 3.4 \times 10^{-2} \text{ M/atm}\)

Solve:  
\[
P = \frac{S}{k_H} = \frac{0.12 \text{ M}}{3.14 \times 10^{-2} \text{ M} \cdot \text{atm}^{-1}} = 3.5 \text{ atm}
\]

Check:  
the unit is correct, the pressure higher than 1 atm meets our expectation from general experience.
Solubility of Ionic Compounds

- The solubility of ionic compounds in water varies tremendously from one solid to another.
- Two forces must be balanced: solid going to water / staying in solid.
  - The force of attraction between water molecules and ions: the stronger the force, the greater the tendency toward solution.
  - The force of attraction between oppositely charged ions: the stronger the force, the more likely the solute will stay in the solid state.

10 minutes break
Colligative properties are those that depend on the concentration of particles in a solution, not on the nature of those particles. These are *colligative* properties

- Freezing Point Depression
- Boiling Point Elevation
- Osmotic Pressure
- Vapor Pressure Lowering

*Jacobus Henricus van 't Hoff*  
(1852-1911)
How do you get from this...

...to this?

Lower Freezing Point

Why?
1. $\Delta T_f$ - Freezing Point Lowering

When a solution of a nonvolatile solute is cooled, it does not freeze until a temperature below the freezing point of the pure solvent is reached.

- The difference in temperature is called the freezing point lowering.
- $T_f$ is the freezing point of the solution.
- $T_f^\circ$ is the freezing point of the pure solvent.

$$\Delta T_f = T_f - T_f^\circ$$
Each mole of solute particles lowers the freezing point of 1 kilogram of water by 1.86 degrees Celsius.

\[ \Delta T = i \cdot K_f \cdot m_{\text{solute}} \]

- \( K_f = 1.86 \, ^\circ\text{C} \cdot \text{kilogram/mol} \)
- \( m = \text{molality of the solution} \)
- \( i = \text{van’t Hoff factor} \)
## K\textsubscript{f} Freezing Point Constant

### Table 10.2: Molal Freezing Point and Boiling Point Constants

<table>
<thead>
<tr>
<th>Solvent</th>
<th>(fp) (°C)</th>
<th>(k_f) (°C/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.00</td>
<td>1.86</td>
</tr>
<tr>
<td>Acetic acid</td>
<td>16.66</td>
<td>3.90</td>
</tr>
<tr>
<td>Benzene</td>
<td>5.50</td>
<td>5.10</td>
</tr>
<tr>
<td>Cyclohexane</td>
<td>6.50</td>
<td>20.2</td>
</tr>
<tr>
<td>Camphor</td>
<td>178.40</td>
<td>40.0</td>
</tr>
<tr>
<td>(p)-Dichlorobenzene</td>
<td>53.1</td>
<td>7.1</td>
</tr>
<tr>
<td>Naphthalene</td>
<td>80.29</td>
<td>6.94</td>
</tr>
</tbody>
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Table of Molal Boiling Point Elevation/Freezing Point Depression Constants for Various Solvents

<table>
<thead>
<tr>
<th>Solvent</th>
<th>Normal BP (°C)</th>
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<tr>
<td>(H_2O)</td>
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<td>0.52</td>
<td>0.0</td>
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<tr>
<td>Ethanol, (C_2H_5OH)</td>
<td>78.4</td>
<td>1.22</td>
<td>-114.6</td>
<td>1.99</td>
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Example 12.8  Freezing Point Depression

Calculate the freezing point of a 1.7 m aqueous ethylene glycol solution

Sort
You are given the molality of a solution and asked to find its freezing point.

Given: 1.7 m solution
Find: freezing point (from $\Delta T_f$)

\[ \Delta T_f = m \times K_f \]

\[ \Delta T_f = 1.7 \text{m} \times 1.86 \degree\text{C/m} \]
\[ = 3.2 \degree\text{C} \]

Freezing point = 0.00 \degree\text{C} – 3.2 \degree\text{C}
\[ = -3.2 \degree\text{C} \]
2. $T_b$ - Boiling Point Elevation

When a solution of a nonvolatile solute is heated, it does not boil until the temperature exceeds the boiling point of the pure solvent.

- The difference in temperature is called the boiling point elevation.
- $T_b$ is the boiling point of the solution.
- $T_b^\circ$ is the boiling point of the pure solvent.

$$\Delta T_b = T_b - T_b^\circ$$
Summary - Boiling and Freezing Point Alteration

• Boiling point elevation and freezing point lowering are both colligative properties
  – Depend on the concentration of the solute in molality
  – Both follow the same dependence:

If \( i = 1 \), then

\[
\Delta T_f = mk_f
\]

\[
\Delta T_b = mk_b
\]

– \( K_f \) is the freezing point lowering constant
– \( K_b \) is the boiling point elevation constant

For water,

\[
k_f = 0.52 \, \frac{^\circ C}{m}
\]

\[
k_b = 1.86 \, \frac{^\circ C}{m}
\]
Boiling Point Elevation

Each mole of solute particles raises the boiling point of 1 kilogram of water by 0.51 degrees Celsius.

\[ \Delta T = i \cdot K_b \cdot m_{solute} \]

- \( K_b = 0.51 \, ^\circ C \cdot \text{kilogram/mol} \)
- \( m = \text{molality of the solution} \)
- \( i = \text{van't Hoff factor} \)

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Example 12.9  Boiling Point Elevation

What mass of ethylene glycol (C\(_2\)H\(_6\)O\(_2\)), in grams, must be added to 1.0 kg of water to produce a solution that boils at 105.0 °C?

Sort
You are given the desired boiling point of an ethylene glycol solution containing 1.0 kg of water and asked to find the mass of ethylene glycol you need to add to achieve the boiling point.

Given: \(\Delta T_b = 5.0 \, ^\circ\text{C}\), 1.0 kg H\(_2\)O

Find: g C\(_2\)H\(_6\)O\(_2\)

\[
\Delta T_b = m \times K_b
\]

\[
m = \frac{\Delta T_b}{K_b} = \frac{5.0 \, ^\circ\text{C}}{0.512 \, ^\circ\text{C}/m} = 9.77m
\]

\[
1.0 \, \text{kg H}_2\text{O} \times \frac{9.77 \, \text{mol C}_2\text{H}_6\text{O}_2}{\text{kg H}_2\text{O}} \times \frac{62.07 \, \text{g C}_2\text{H}_6\text{O}_2}{\text{1 mol C}_2\text{H}_6\text{O}_2} = 6.1 \times 10^2 \, \text{g C}_2\text{H}_6\text{O}_2
\]
The van't Hoff Factor, $i$

Solutions may have two, three or more times the effect on boiling point, freezing point, and osmotic pressure, depending on its solute dissociation.

NOTE: $i =$ van’t Hoff factor, which is determined experimentally and represents the degree of dissociation of the solute in the solvent.

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Dissociation Equations and the Determination of $i$

- $\text{NaCl(s)} \rightarrow \text{Na}^+(\text{aq}) + \text{Cl}^-(\text{aq})$
- $\text{AgNO}_3(\text{s}) \rightarrow \text{Ag}^+(\text{aq}) + \text{NO}_3^-(\text{aq})$
- $\text{MgCl}_2(\text{s}) \rightarrow \text{Mg}^{2+}(\text{aq}) + 2 \text{Cl}^-(\text{aq})$
- $\text{Na}_2\text{SO}_4(\text{s}) \rightarrow 2 \text{Na}^+(\text{aq}) + \text{SO}_4^{2-}(\text{aq})$
- $\text{AlCl}_3(\text{s}) \rightarrow \text{Al}^{3+}(\text{aq}) + 3 \text{Cl}^-(\text{aq})$

$i = 2$

$i = 3$

$i = 3$

$i = 4$
Boiling-Point Elevation and Freezing-Point Depression

What is the freezing point (in °C) of a solution prepared by dissolving 7.40 g of MgCl$_2$ in 110 g of water? The van’t Hoff factor for MgCl$_2$ is $i = 2.7$.

Calculate the moles of MgCl$_2$:

$$\frac{7.40 \text{ g}}{95.2 \text{ g}} \times \frac{1 \text{ mol}}{1 \text{ mol}} = 0.07777 \text{ mol}$$

Calculate the molality of the solution:

$$\frac{0.07777 \text{ mol}}{110 \text{ g}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 0.71 \text{ mol/kg}$$
Boiling-Point Elevation and Freezing-Point Depression

Calculate the freezing point of the solution:

\[ \Delta T_f = K_f m i = 1.86 \frac{^\circ C \text{ kg}}{\text{mol}} \times 0.71 \frac{\text{mol}}{\text{kg}} \times 2.7 = 3.6 \, ^\circ C \]

\[ T_f = 0.0 \, ^\circ C - 3.6 \, ^\circ C = -3.6 \, ^\circ C \]

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Ideal vs. Real van’t Hoff Factor

The ideal van’t Hoff Factor is only achieved in **VERY DILUTE** solution.

<table>
<thead>
<tr>
<th>Solute</th>
<th>1.0</th>
<th>0.10</th>
<th>0.010</th>
<th>0.0010</th>
<th>...</th>
<th>Inf dil*</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaCl</td>
<td>1.81</td>
<td>1.87</td>
<td>1.94</td>
<td>1.97</td>
<td>...</td>
<td>2</td>
</tr>
<tr>
<td>MgSO₄</td>
<td>1.09</td>
<td>1.21</td>
<td>1.53</td>
<td>1.82</td>
<td>...</td>
<td>2</td>
</tr>
<tr>
<td>Pb(NO₃)₂</td>
<td>1.31</td>
<td>2.13</td>
<td>2.63</td>
<td>2.89</td>
<td>...</td>
<td>3</td>
</tr>
</tbody>
</table>

Limit to applicability of these laws usually is 1m
3. Osmosis and Semi-Permeable Membranes

- Consider the concentration of solvent in a solution
  - Concentration is lower than it is for the pure solvent
  - Solvent will flow from an area of high concentration to an area of low concentration
  - Water will flow from high concentration to low
  - Process is called *osmosis*

The necessary pressure to stop osmotic flow is called \( \text{osmotic pressure} (\pi) \) of the solution.
Osmotic Pressure

The minimum pressure that stops the osmosis is equal to the osmotic pressure of the solution.
Osmotic Pressure Calculations

\[ \Pi = iMRT \]

\( \Pi \) = Osmotic pressure

\( i \) = van’t Hoff Factor

\( M \) = Molarity of the solution

\( R \) = Gas Constant = 0.08206 L·atm/mol·K
Example 12.10  Osmotic Pressure

The osmotic pressure of a solution containing 5.87 mg of an unknown protein per 10.0 mL of solution is 2.45 torr at 25 °C. Find the molar mass of the unknown protein.

Sort
You are given that a solution of an unknown protein contains 5.87 mg of the protein per 10.0 mL of solution. You are also given the osmotic pressure of the solution at a particular temperature and asked to find the molar mass of the unknown protein.

Given: 5.87 mg protein
10.0 mL solution
Π = 2.45 torr
T = 25 °C
Find: molar mass of protein (g/mol)

\[ \Pi = \text{MRT} \]

\[ M = \frac{\Pi}{RT} = \frac{2.45 \text{ torr} \times \frac{1 \text{ atm}}{760 \text{ torr}}}{0.08206 \ \text{L} \cdot \text{atm} / \text{mol} \cdot \text{K} (298 \text{ K})} \]

\[ = 1.318 \times 10^{-4} \text{ M} \]

\[ 10.0 \ \text{mL} \times \frac{1 \text{ L}}{1000 \ \text{mL}} \times \frac{1.318 \times 10^{-4} \text{ mol}}{1 \text{ L}} \]

\[ = 1.318 \times 10^{-6} \text{ mol} \]

Molar mass = \( \frac{\text{mass protein}}{\text{moles protein}} \)

\[ = \frac{5.87 \times 10^{-3} \text{ g}}{1.318 \times 10^{-6} \text{ mol}} = 4.45 \times 10^3 \text{ g/mol} \]
Osmolarity

- Solutions of the same osmotic pressure are said to be isotonic; i.e., they have equal osmolarity
  - Important to medical applications
  - Consider a red blood cell
    - If the concentration of ions is larger inside the cell, water will flow in, causing the cell to burst (hemolysis)
    - If the concentration of ions is larger outside the cell, water will flow out, causing the cell to shrivel (crenation)
    - Solutions such as intravenous fluid are prepared to be osmolar with the blood plasma to prevent hemolysis or crenation from occurring
An isosmotic solution has the same osmotic pressure as the solution inside the cell; as a result there is no net flow of water into or out of the cell.

(a) Normal red blood cells
(b) Shriveled red blood cells
(c) Swollen red blood cells
4. Colligative Properties - **Vapor Pressure Lowering**
Colligative Properties – Vapor Pressure

• Step 1: A solvent in a closed container reaches a state of dynamic equilibrium.

• Step 2: The pressure exerted by the vapor in the headspace is referred to as the vapor pressure of the solvent.

• The addition of any nonvolatile solute (one with no measurable vapor pressure) to any solvent reduces the vapor pressure of the solvent.
Colligative Properties – Vapor Pressure

• Nonvolatile solutes reduce the ability of the surface solvent molecules to escape the liquid.
  – Vapor pressure is reduced.

• The extent of vapor pressure lowering depends on the amount of solute.
  – Raoult’s Law quantifies the amount of vapor pressure lowering that is observed.
Colligative Properties – Vapor Pressure

• Raoult’s Law:

\[ P_A = X_A P^o_A \]

where \( P_A \) = partial pressure of the solvent (A) vapor above the solution (ie with the solute)

\( X_A \) = mole fraction of the solvent (A)

\( P^o_A \) = vapor pressure of the pure solvent (A)
The vapor pressure of pure water at 25 °C is 23.76 mm Hg. What is the vapor pressure of a solution made from 1.00 mol glucose in 15.0 mol of water at 25 °C? Glucose is a nonvolatile solute.

\[
P_{\text{soln}} = P_{\text{solv}} X_{\text{solv}}
\]

\[
= 23.76 \text{ mm Hg} \times \frac{15.0 \text{ mol}}{1.00 \text{ mol} + 15.0 \text{ mol}} = 22.3 \text{ mm Hg}
\]
Example 12.12 Calculating the Vapor Pressure of a Solution Containing an Ionic Solute

A solution contains 0.102 mol Ca(NO$_3$)$_2$ and 0.927 mol H$_2$O. Calculate the vapor pressure of the solution at 55 °C. The vapor pressure of pure water at 55 °C is 118.1 torr. (Assume that the solute completely dissociates.)

Sort

You are given the number of moles of each component of a solution and asked to find the vapor pressure of the solution. You are also given the vapor pressure of pure water at the appropriate temperature.

Given: 0.102 mol Ca(NO$_3$)$_2$
0.927 mol H$_2$O
$P_{H_2O} = 118.1$ torr (at 55 °C)

Find: $P_{solution}$

\[
\begin{align*}
\text{Ca(NO}_3\text{)}_2(s) & \rightarrow \text{Ca}^{2+}(aq) + 2 \text{NO}_3^-(aq) \\
\chi_{H_2O} &= \frac{n_{H_2O}}{3 \times n_{Ca(NO}_3\text{)}_2 + n_{H_2O}} \\
&= \frac{0.927 \text{ mol}}{3 \times 0.102 \text{ mol} + 0.927 \text{ mol}} \\
&= 0.7518
\end{align*}
\]

\[
\begin{align*}
P_{solution} &= \chi_{H_2O} P_{H_2O} \\
&= 0.7518 \times 118.1 \text{ torr} \\
&= 88.8 \text{ torr}
\end{align*}
\]
SOLUTION COLLAGIGATIVE PROPERTIES

- Depends on the collective effect of the number of solute particles but not on the nature of the solute. Van Hoff’s factor might be needed,

1. Boiling point elevation
   \[ \Delta T_b = k_b \, M \]

2. Freezing point depression
   \[ \Delta T_f = -k_f \, M \]

3. Vapor pressure lowering
   Raoult’s law \( P_a = X_A P^o_A \)

4. Osmotic Pressure
   \[ \Pi = MRT = \left( \frac{N}{V} \right)RT \]
EXERCISE: Molar Mass from Freezing-Point Depression

A solution of an unknown nonvolatile electrolyte was prepared by dissolving 0.250 g of the substance in 40.0 g of CCl₄. The boiling point of the resultant solution was 0.357°C higher than that of the pure solvent. Calculate the molar mass of the solute. For pure CCl₄ \( k_b = 5.02 \, ^\circ C/\text{m} \)

**Answer:**

\[
\text{molality} = \frac{n_{\text{solute}} \, \text{mol}}{40.0 \times 10^{-3} \, \text{kg}} = \frac{\Delta T_b}{k_b} = \frac{0.375 \, ^\circ C}{5.02 \, ^\circ C/\text{m}^{-1}} = 0.0711 \text{m}
\]

\[
\frac{n_{\text{solute}}}{40.0 \times 10^{-3} \, \text{kg}} = 0.0711 \text{m}
\]

\[
n_{\text{solute}} = 40.0 \times 10^{-3} \, \text{kg} \times 0.0711(\text{mol.kg}^{-1}) = 2.84 \times 10^{-3} \, \text{mol of solute}
\]

\[
n_{\text{solute}} = \frac{m}{\text{MM}} = 2.84 \times 10^{-3} \, \text{mol of solute}
\]

\[
\therefore \text{MM} = \frac{m (\text{g})}{n_{\text{solute}} (\text{mol})} = \frac{0.250 \text{g}}{2.84 \times 10^{-3} \, \text{mol}} = 88.0 \, \text{g.mol}^{-1}
\]
**EXERCISE:** Molar Mass from Osmotic Pressure

The osmotic pressure of an aqueous solution of a certain protein was measured in order to determine the protein’s molar mass. The solution contained 3.50 mg of protein dissolved in sufficient water to form 5.00 mL of solution. The osmotic pressure of the solution at 25°C was found to be 1.54 torr. Calculate the molar mass of the protein.

**Answer**

\[
\pi V = nRT
\]

\[
n = \frac{\pi V}{RT}
\]

\[
= \frac{1.54 \text{ mmHg}}{0.0821 \text{ L.atm.mol}^{-1}.\text{K}^{-1} \times 298 \text{ K}} \times \frac{1 \text{ atm}}{760 \text{ mmHg}} \times 5 \text{ ml} \times \frac{1 \text{ L}}{1000 \text{ ml}} = 4.14 \times 10^{-7} \text{ mol}
\]

\[
n = \frac{m}{MM} \Rightarrow MM = \frac{m}{n} = \frac{3.50 \times 10^{-3} \text{ g}}{4.14 \times 10^{-7} \text{ mol}} = 8.45 \times 10^{3} \text{ g.mol}^{-1}
\]